Eiffel: Inferring Input Ranges of Significant Floating-point Errors via Polynomial Extrapolation

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Outline

- What is floating-point error?
- Error detection is important
- Existing approaches

BACKGROUND

DIFFICULTIES

APPROACH

EVALUATION

Floating-point Errors

- Some inputs may trigger significant FP errors
- Consider:

$$f(x) = \frac{\tan(x) - \sin(x)}{x^3} \qquad \lim_{x \to 0} f(x) = 0.5$$

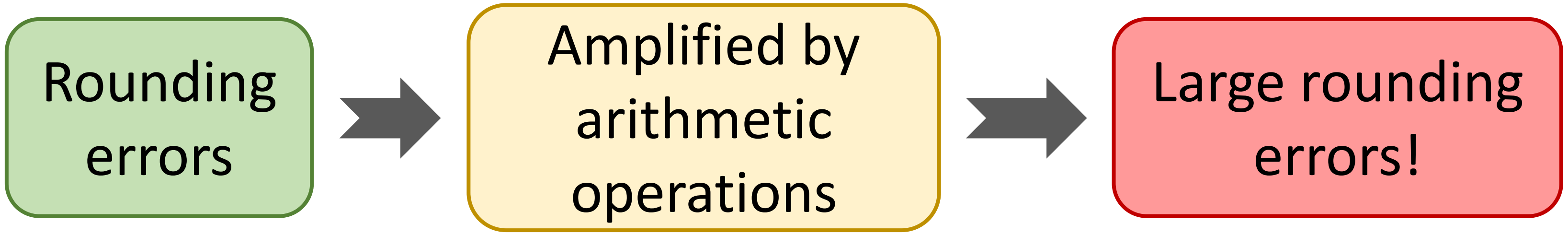
```
double f(double x) {
    double num = tan(x) – sin(x);
    double den = x * x * x;
    return num / den;
}
```

```
>>> f(1e-7) // 64 bits result
0.5029258124322410

Accurate result //128 bits result
0.5000000000000012
```

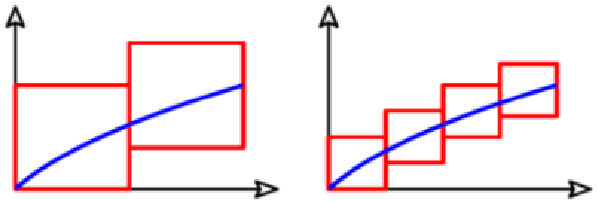
Error Detection is Crucial

- FP errors are infamous problem in software development



- Large rounding errors may lead to catastrophic software failures
  - Missile yaw *[Skeel '92]*
  - Stock trading disorder *[Quinn '83]*
  - Rocket launch failure *[Lions '96]*

Existing approaches



Static analysis

- Abstract interpretation
- Symbolic execution
- Interval arithmetic
- Affine arithmetic
- …

Goal: Approximate error bounds

- Over-approximated

Dynamic analysis

- Random search
- Binary guided random testing (BGRT)
- Atomic condition
- …

Goal: Find the maximal error

+ Real error

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- What is guided search?
- Difficulties for guided searches

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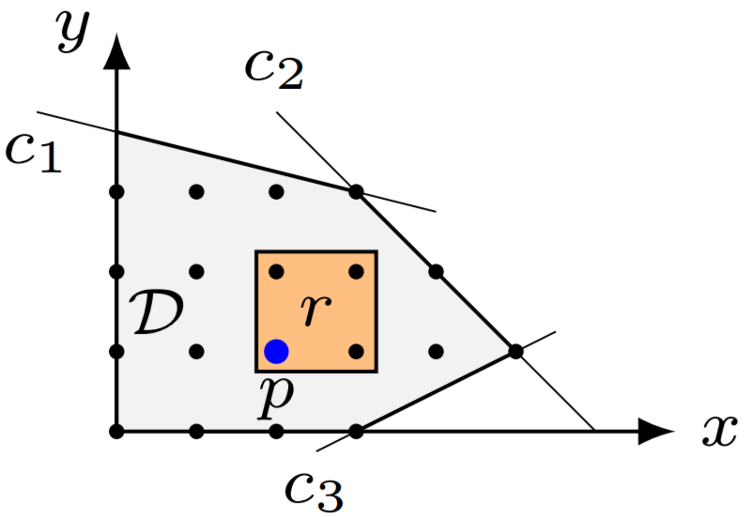
Difficulties for guided searches



Guided search

Process

- Search space: $D = \{(x, y) : c_1 \wedge c_2 \wedge c_3\}$
- c_1, c_2, c_3 are constraints of the two variables x and y;
- The points within r are input values that may trigger significant errors;
- p is the input that triggers the maximal error.

Goal: Find the point p

Difficulties

- D may be complex and large
- r and p could both be many

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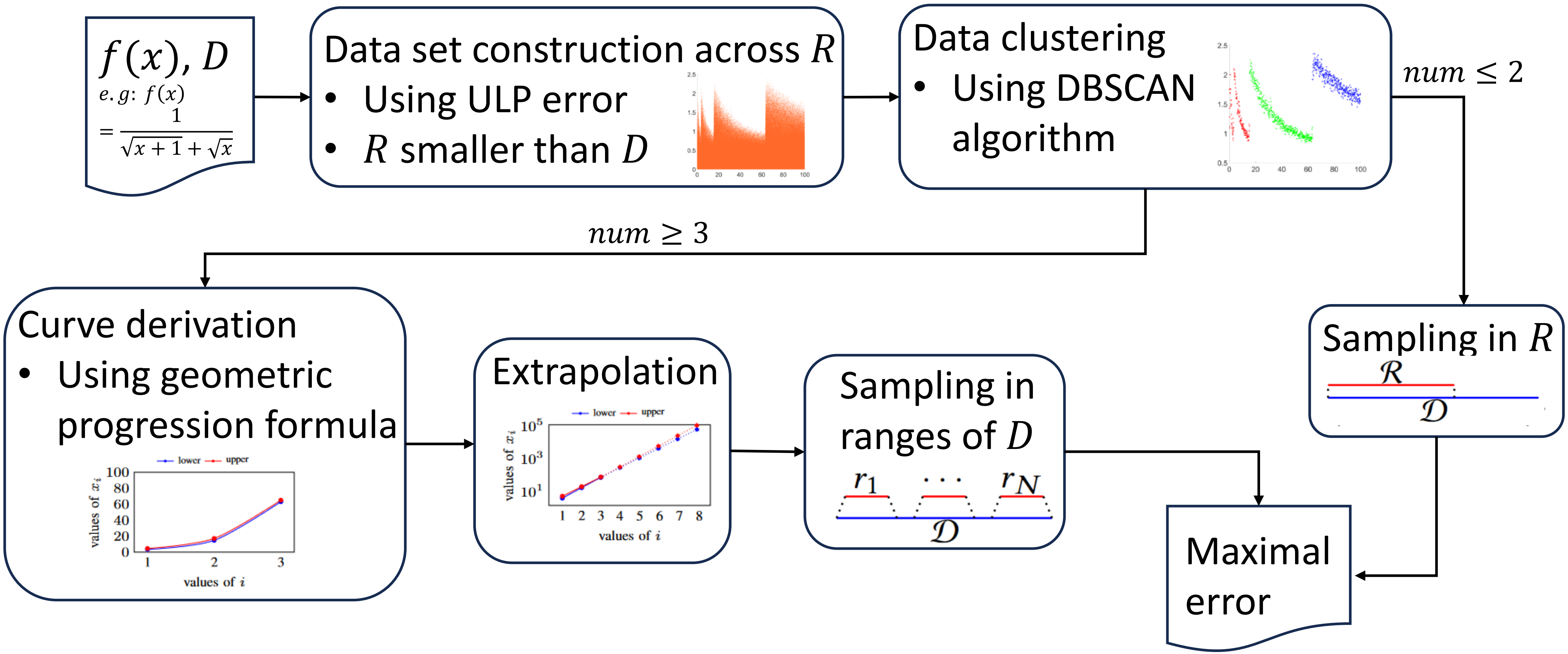
- Error analysis using EIFFEL
- How EIFFEL works?

Error analysis using EIFFEL

Core idea of EIFFEL: Inferring error-inducing ranges instead of searching them in D



Data set construction

Two issues

- Determine R
  - FP numbers are non-uniformly distributed
    - [-1,1]   49.95% (double type)
    - Dense near 0

$\Longrightarrow$ Small interval as close to 0

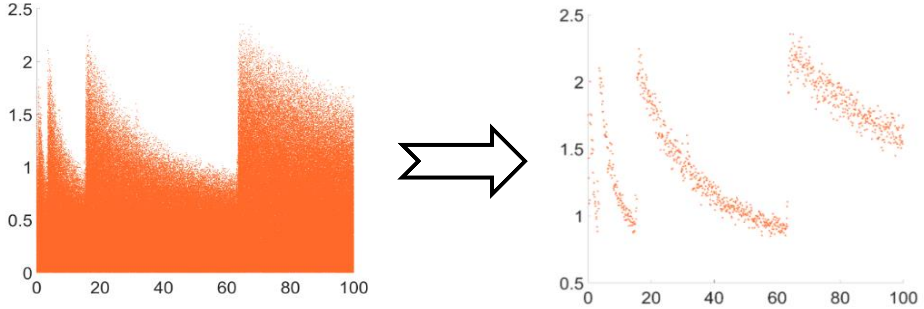
- Deciding the number of input values s to compute the ULP errors, considers
  - Performance $\Longrightarrow s = 500{,}000$ (0.17 seconds)
  - Accurate error distribution $\Longrightarrow$ Uniformly distributed inputs

Boundary extraction and data clustering

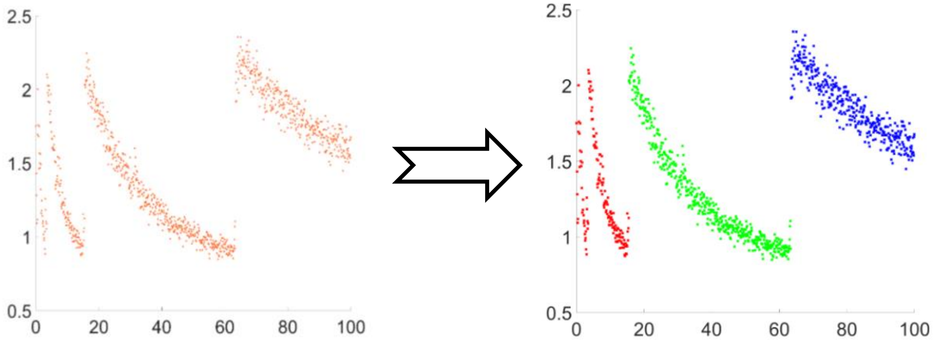
- Boundary extraction
  - Reduce the number of data points to accelerate clustering

  Step 1: Gather every $g = 500$ samples in one group

  Step 2: Preserve the one that has the largest ULP error in every group



- Data clustering
  - Obtain the maximal error point for each cluster for fitting the function
    - DBSCAN clustering algorithm



Curve derivation and polynomial extrapolation

- Curve derivation
  - Two challenges
    - Each peak point is a 2D coordinate
    - Extrapolated points may still follow the same distribution as that of the stars
  - Solution
    - 2D coordinates into 1D form

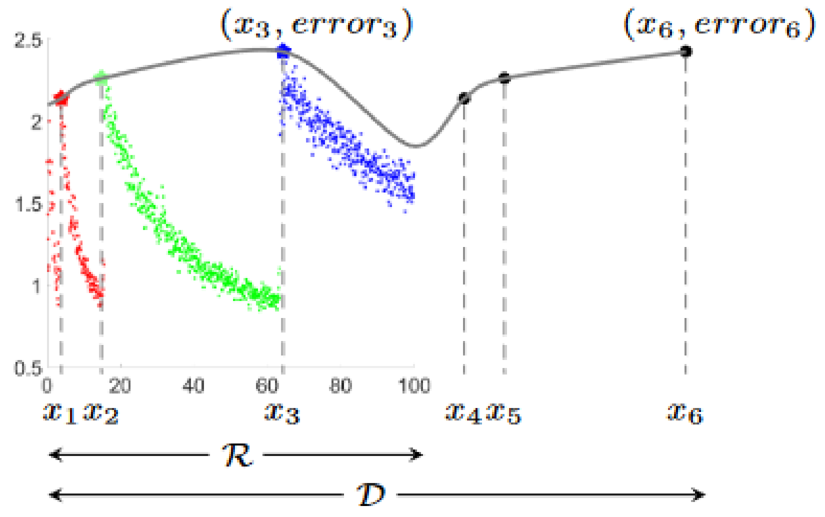
$(x_1, error_1), (x_2, error_2) \dots (x_i, error_i) \rightarrow (1, x_1), (2, x_2) \dots (i, x_i)$

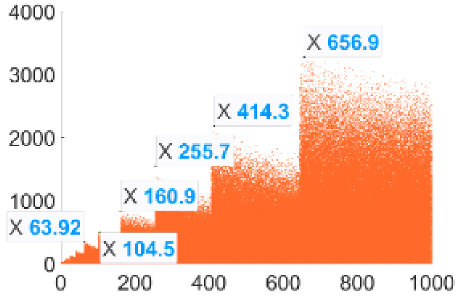
    - Assume the x_i form a geometric sequence or a geometric progression

$$x_i = x_1 \times q^{i-1} \qquad i \geq 1$$

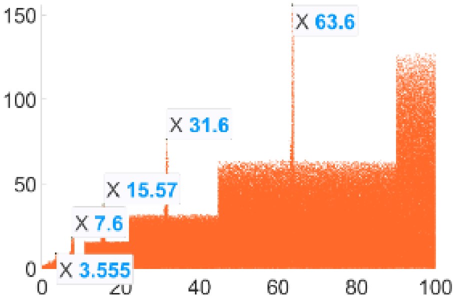
The assumption is observed from extensive experiments

- Derive two curves
  - Best fit for lower bound $x_i - r$ and upper bound $x_i + r$ that covers each x_i





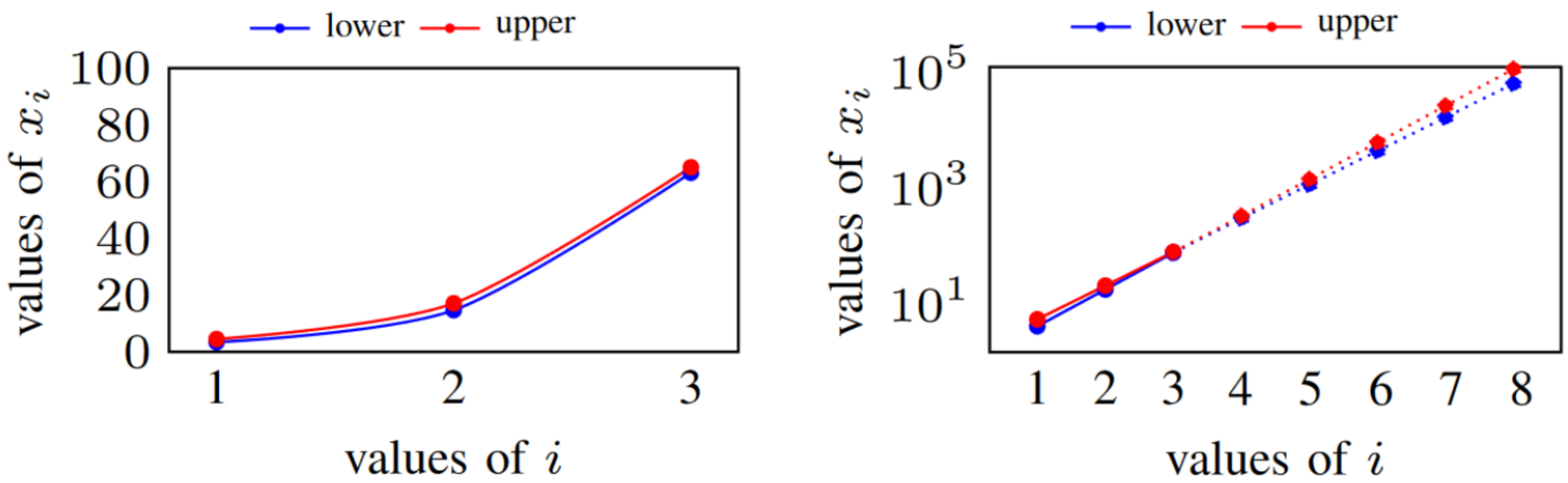
(a) NMSEexample36; $q \approx 1.59$.



(b) NMSEproblem331; $q \approx 2.03$.

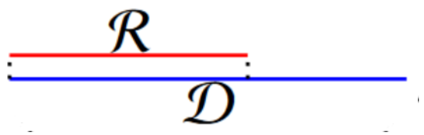
Curve derivation and polynomial extrapolation

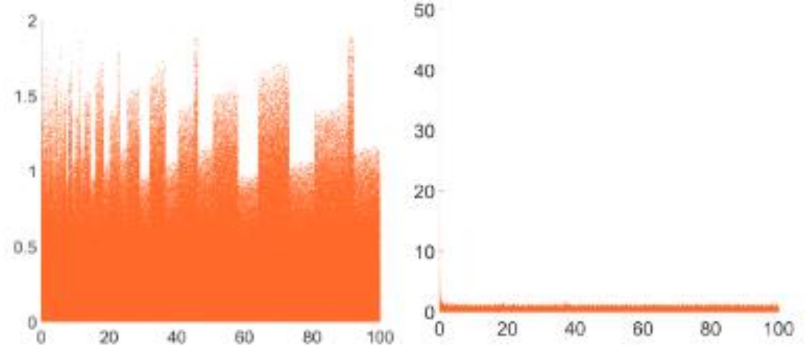
- Polynomial extrapolation



Error detection

- $num = 1$ or $num = 2$
  - Return maximal error across R

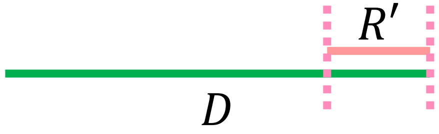




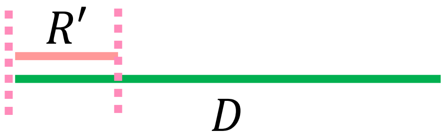
$num = 1$ stable

$num = 1$ or 2 unstable

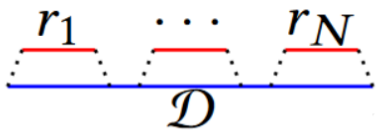
- $num \geq 3$
  - Consider the monotonicity
    - $error_1 < error_2 < \cdots$



    - $error_1 > error_2 > \cdots$



  - Polynomial extrapolation



Generalization for multi-variate scenarios

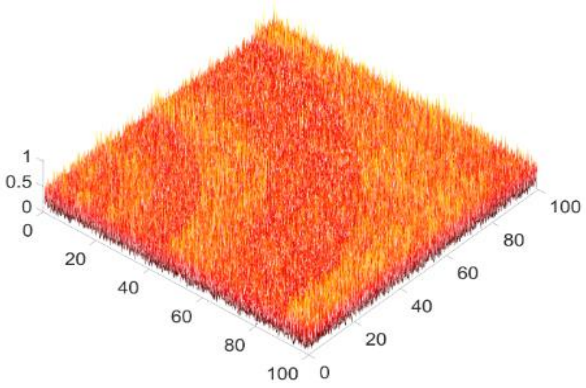
Similar to the single-variate case

- Two adaptations
  - Project the multi-dimensional plot onto the variable space

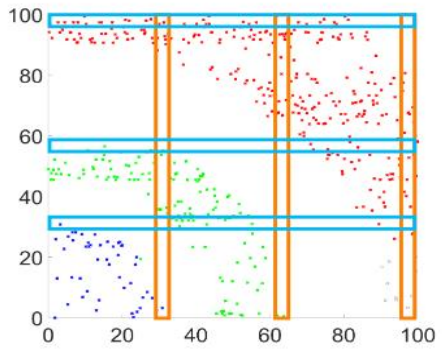
⇒ Produce the variable coordinates of the errors

  - Perform curve fitting along one dimension each time

⇒ Produce (hyper-)rectangular ranges but is still more effective than existing approaches



Error distribution



Projection and clustering

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EVALUATION

- How effective?
- How quality and quantity?
- How overhead?

Evaluation

- Benchmarks: total 70 expressions
  - 66 expressions are from FPBench
  - 4 expressions are from real-life numerical programs

- D is set using large but reasonable ranges

Total Benchmarks	Single-variate	Multi-variate
70	30	40

Evaluation — Effectiveness

- Effectiveness
  - Compared with the state-of-the-art techniques

Techniques	Number of errors detected
EIFFEL	70
S3FP	43
ATOMU	30

ATOMU is only able to report errors for the 30 single-variate examples

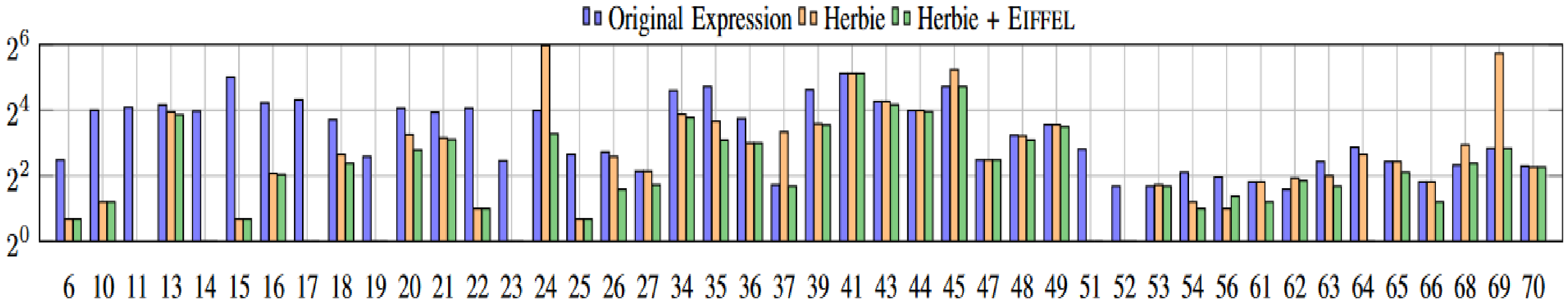
S3FP returns empty results for 27 benchmarks

Evaluation — Inferred input ranges

- Quantity
  - Regina fails to infer input ranges at a large size of D
  - EIFFEL obtains more input ranges than PSAT

- Quality
  - Feed the inferred input ranges to Herbie

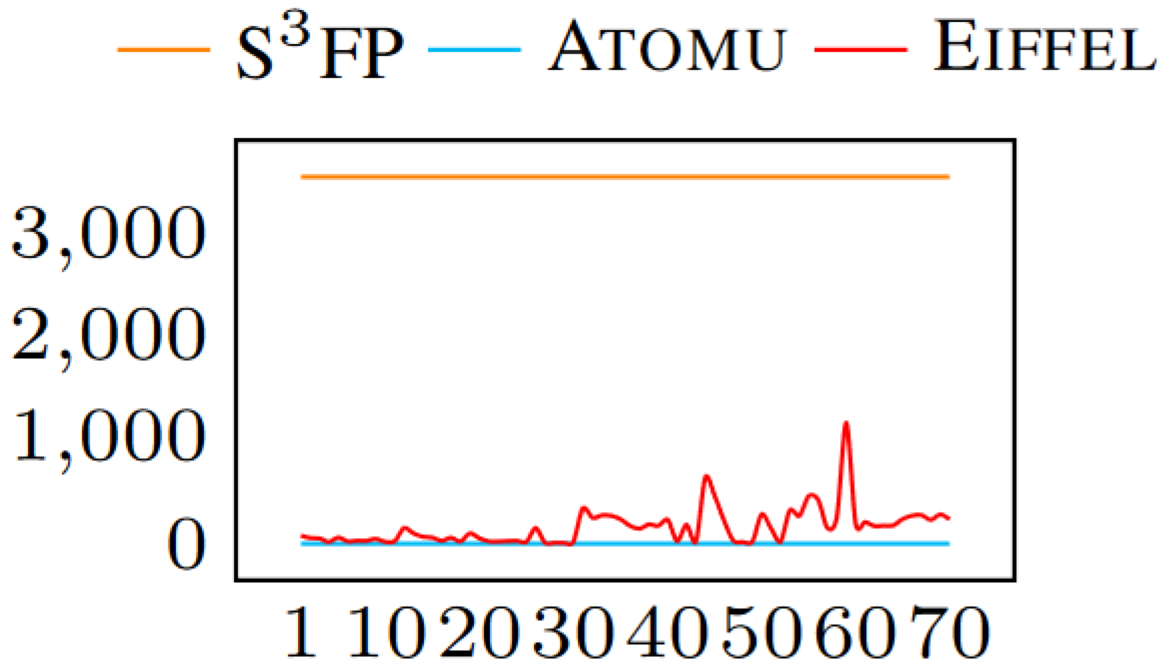
Improvement	Original Herbie version
Average	3.35 bits
Maximal	53.3 bits



Benchmark	Number	
	EIFFEL	PSAT
predatorPrey	15	2
sqrt_add	7	2
verhulst	12	2
nonlin1_test2	14	2
Intro-example	14	2
NMSEexample35	15	5
NMSEexample37	27	4
carbonGas	21	3

Evaluation — Overhead

Time overhead is between ATOMU and S3FP



https://github.com/zuoyanzhang/EIFFEL